Angular distributions and rotations of frames

In vector meson decays to lepton pairs

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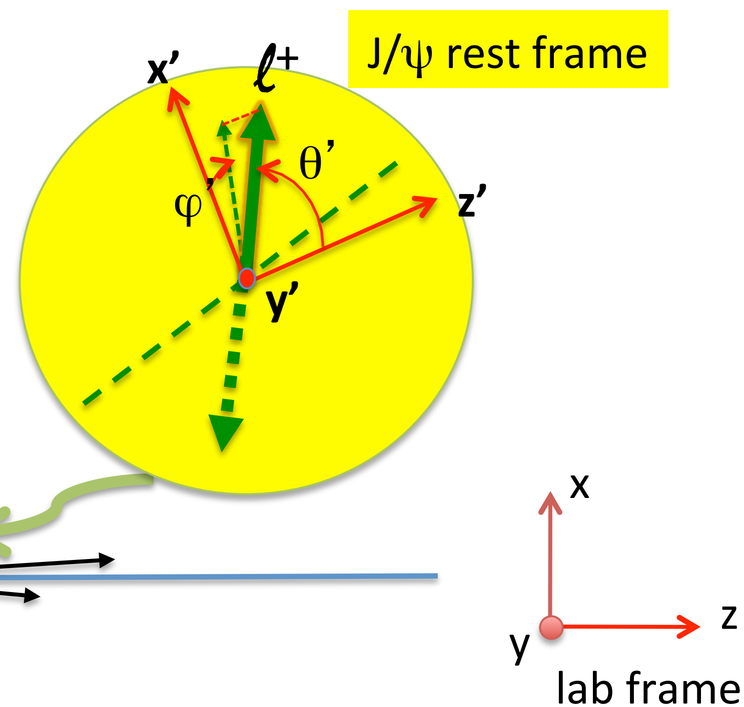
Reference frames, angular variables

The z axis is directed along the beam.
The lab frame is oriented so that the J/ψ is in the x-z plane.

The y and y' axes are parallel.

Different choices are possible for the the direction of z' (and x')



Collision axis

Functional form of the angular distribution

for decays of 1^{--} states to lepton pairs decays

$$dN/d\Omega \propto [\,1 + \lambda_\theta \cos^2(\theta) + \lambda_\varphi \sin^2(\theta)\cos(2\varphi) + \lambda_{\theta\varphi} \sin(2\theta)\cos(\varphi)\,] / (1+\lambda_\theta/3)$$

It is valid in any frame (eg: CS or H, only the y-axis direction is fixed) under general conditions, but of course the three coefficients have different values in different frames:

Rotation of frame as $x' = x\cos(\delta) - z\sin(\delta)$ etc.,

$$1 \Rightarrow 1 + \frac{1}{2}\sin^2(\delta)\cdot(\lambda_\theta - \lambda_\varphi) + \frac{1}{2}\sin(2\delta)\cdot\lambda_{\theta\varphi} = N$$

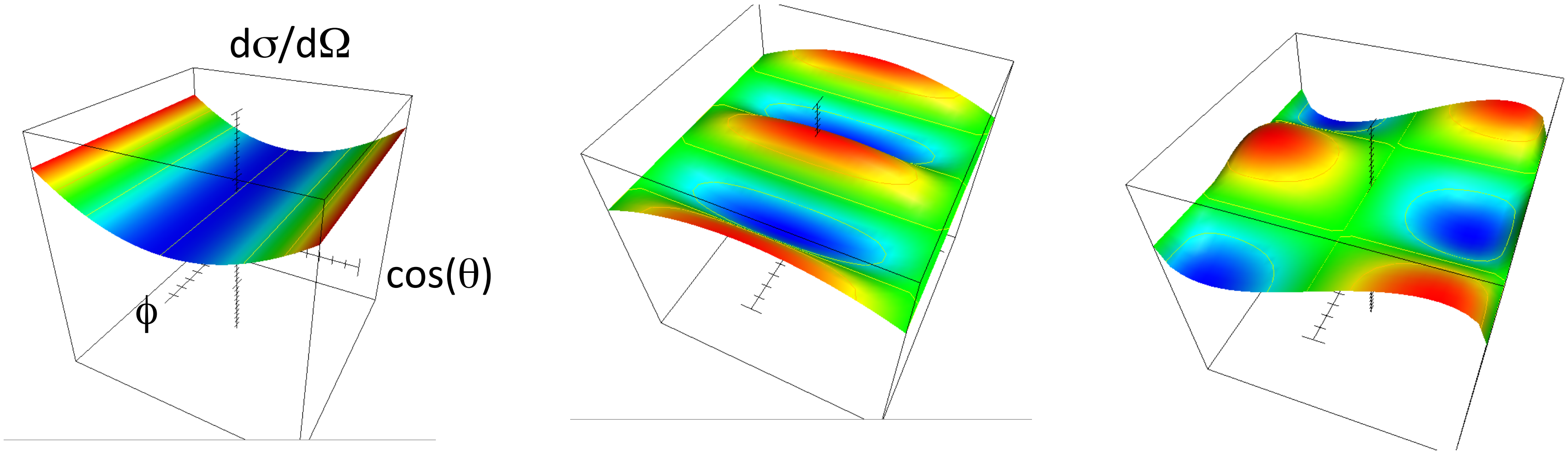
$$\lambda_\theta \Rightarrow \left(1 - \frac{3}{2}\sin^2\delta\right)\lambda_\theta + \frac{3}{2}\sin^2(\delta)\cdot\lambda_\varphi - \frac{3}{2}\sin(2\delta)\cdot\lambda_{\theta\varphi} = N\,\lambda_\theta'$$

$$\lambda_\varphi \Rightarrow \left(1 - \frac{1}{2}\sin^2\delta\right)\lambda_\varphi + \frac{1}{2}\sin^2(\delta)\cdot\lambda_\theta + \frac{1}{2}\sin(2\delta)\cdot\lambda_{\theta\varphi} = N\,\lambda_\varphi'$$

$$\lambda_{\theta\varphi} \Rightarrow \cos(2\delta)\cdot\lambda_{\theta\varphi} + \frac{1}{2}\sin(2\delta)\cdot(\lambda_\theta - \lambda_\varphi) = N\,\lambda_{\theta\varphi}'$$

Between CS and H frames, the rotation angles approximates ~ 90 deg over large areas of ATLAS or CMS acceptance

Modes defined by the coefficients



$\lambda_\theta=+1$, $\lambda_\varphi=0$, $\lambda_{\theta\varphi}=0$ $\lambda_\theta=0$, $\lambda_\varphi=+0.5$, $\lambda_{\theta\varphi}=0$ $\lambda_\theta=0$, $\lambda_\varphi=0$, $\lambda_{\theta\varphi}=+0.5$

Acceptance requirements for angular polarization study:

λ_θ : benefits from large coverage in $\cos(\theta)$,
and coverage in ϕ to disentagle from λ_φ (see below)

λ_φ : needs coverage in ϕ for rather small $|\cos(\theta)|$.

$\lambda_{\theta\varphi}$: needs coverage in ϕ for $|\cos(\theta)| \approx 0.3$-$0.9$

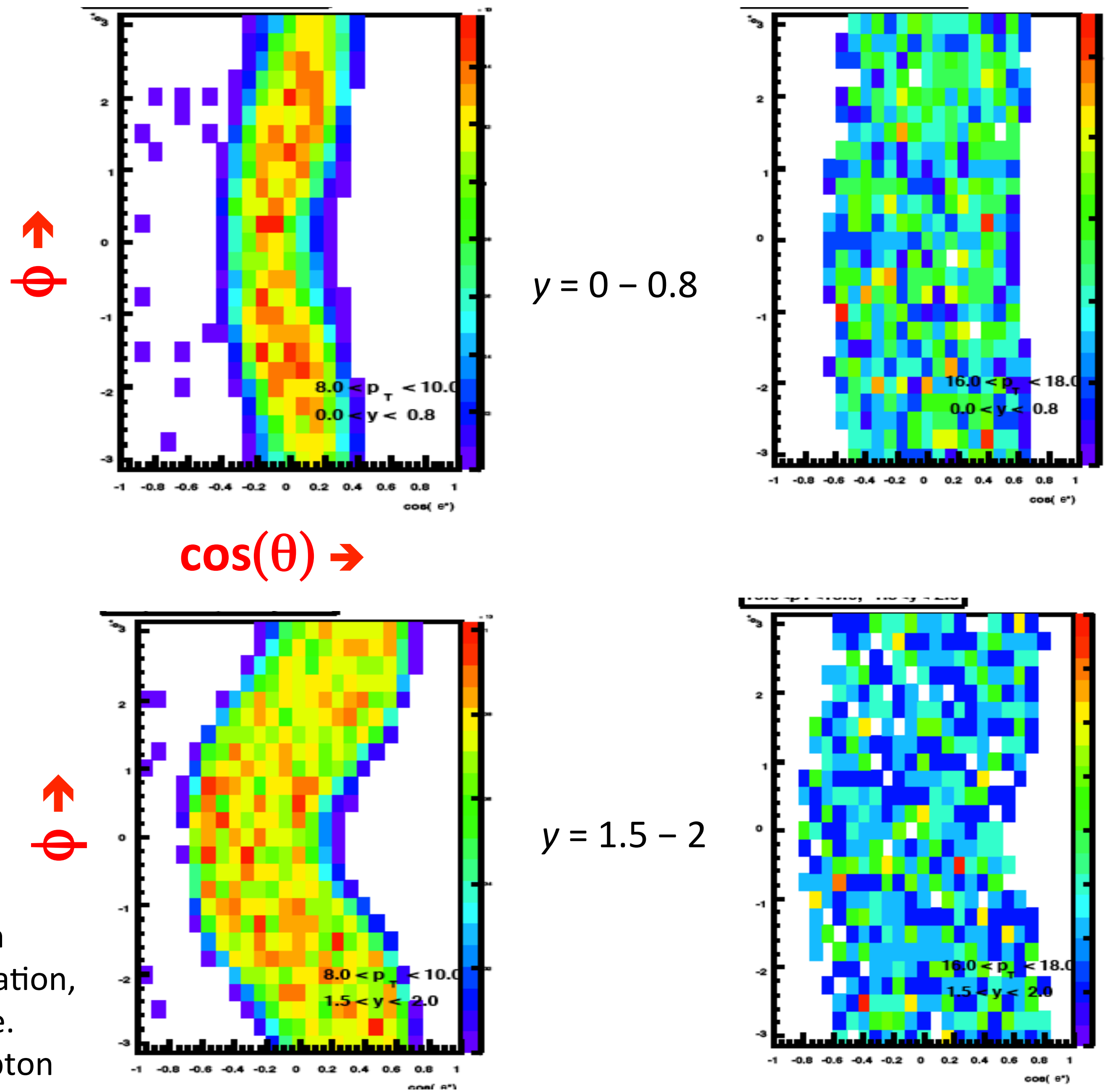
Expected 2D distributions of events in representative bins in *y*, p_T

Helicity frame

λ_φ : coverage already at low p_T

$\lambda_{\theta\varphi}$ and in particular λ_θ: improved coverage in regions of large p_T and/or large rapidity

[Monte Carlo with uniform angular distribution generation, ATLAS-inspired acceptance. Negative y, or opposite lepton charge convention give opposite *curvatures*]



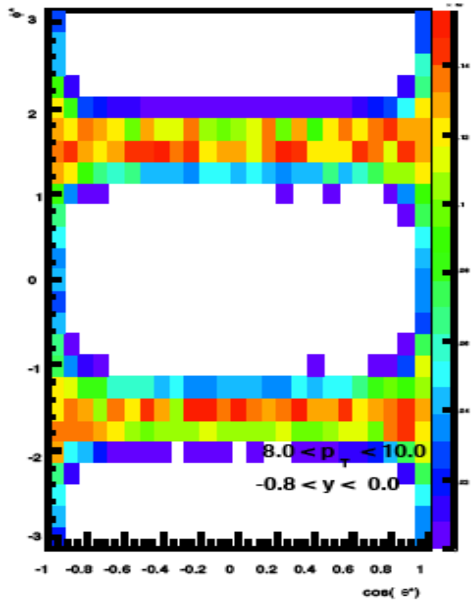
p_T= 8 – 10 GeV/c

p_T= 16 – 18 GeV/c

2D distributions in representative bins in y, p_T

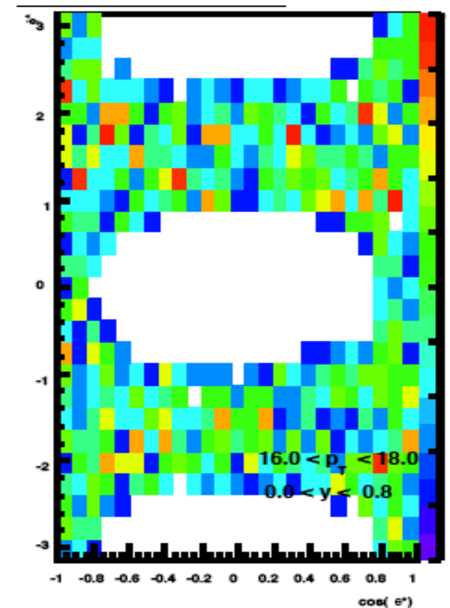
C-S frame

↑ φ



cos(θ) ➔

$y = 0 - 0.8$

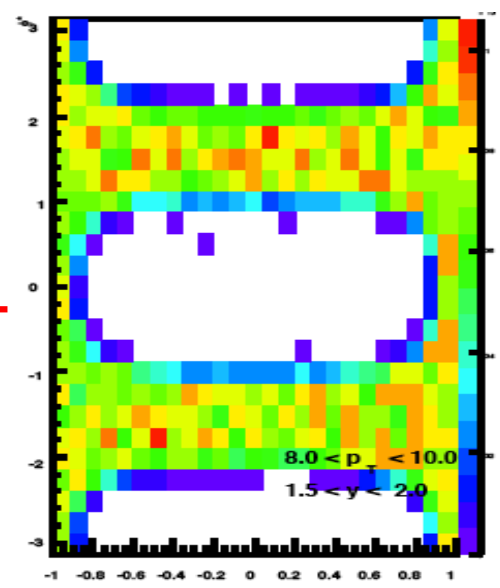


cos(θ) ➔

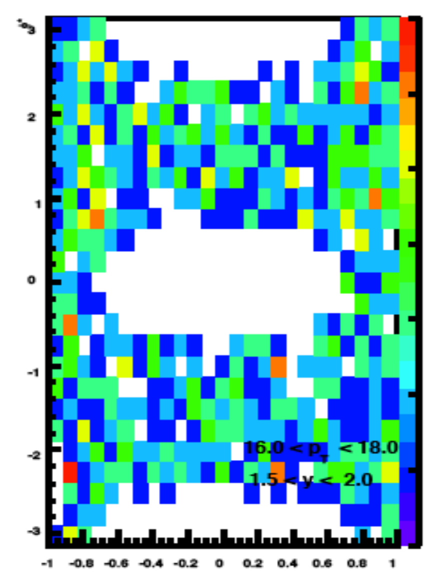
Good coverage for $\lambda_{\theta\varphi}$;
λ_θ and λ_φ entangled at low-moderate p_T/ y :

For $\varphi \approx \pm\pi/2$, we are sensitive to the combination:
$(\lambda_\theta+\lambda_\varphi)/(1-\lambda_\varphi)$
[not useful if $\lambda_\theta \approx -\lambda_\varphi$]

↑ φ



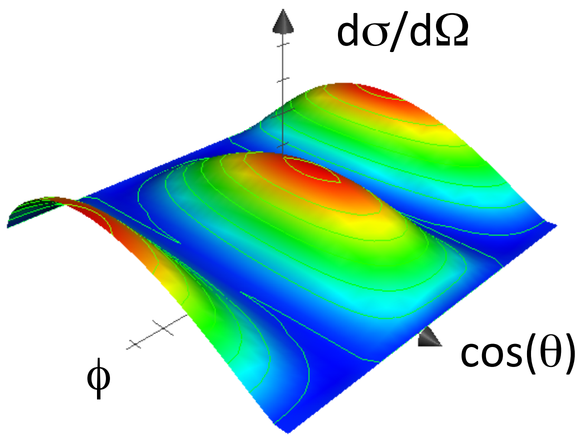
$y = 1.5 - 2$



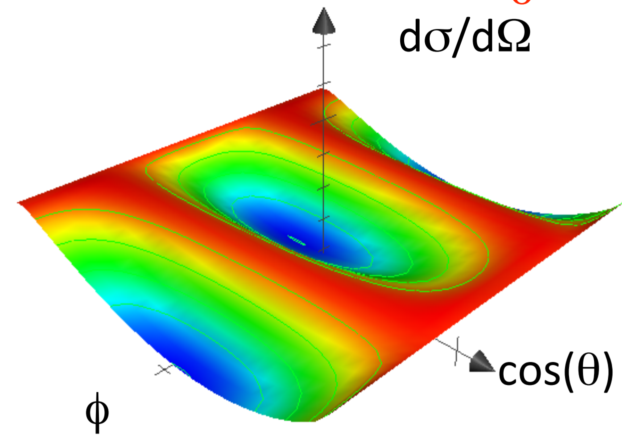
p_T= 8 – 10 GeV/c

p_T= 16 – 18 GeV/c

Examples of angular distributions for $\lambda_\theta + \lambda_\varphi = 0$



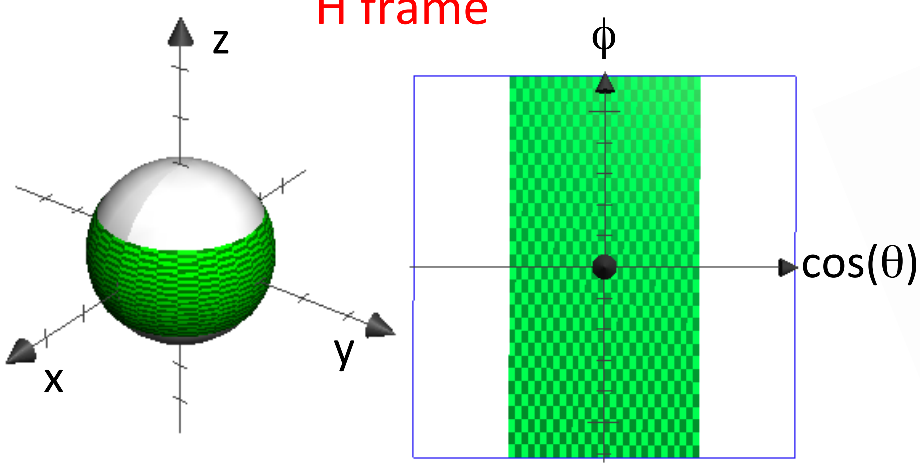
λ_θ=-0.5, λ_φ=+0.5, $\lambda_{\theta\varphi}$=+0.2



λ_θ=+0.5, λ_φ=-0.5, $\lambda_{\theta\varphi}$=+0.2

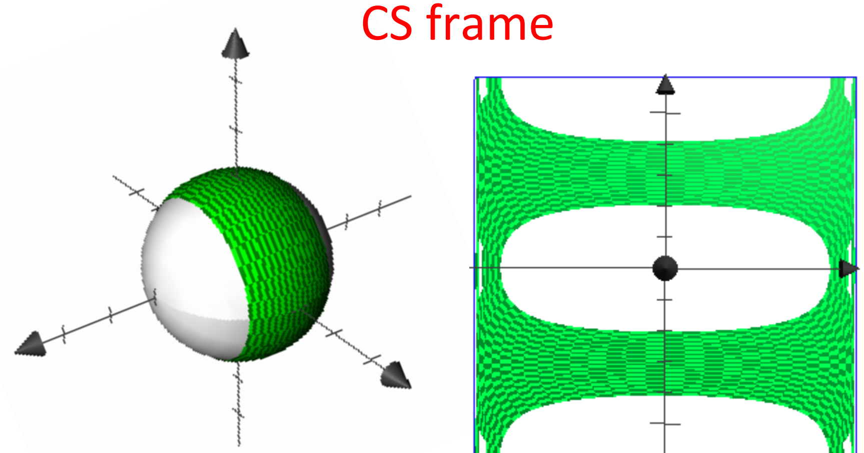
Illustration of acceptance for intermediate p_T, low y

H frame



Approximate acceptance as depending on $|P_{muon}|$

CS frame



Approximate rotation between frames with 90 deg.

General properties of the coefficients

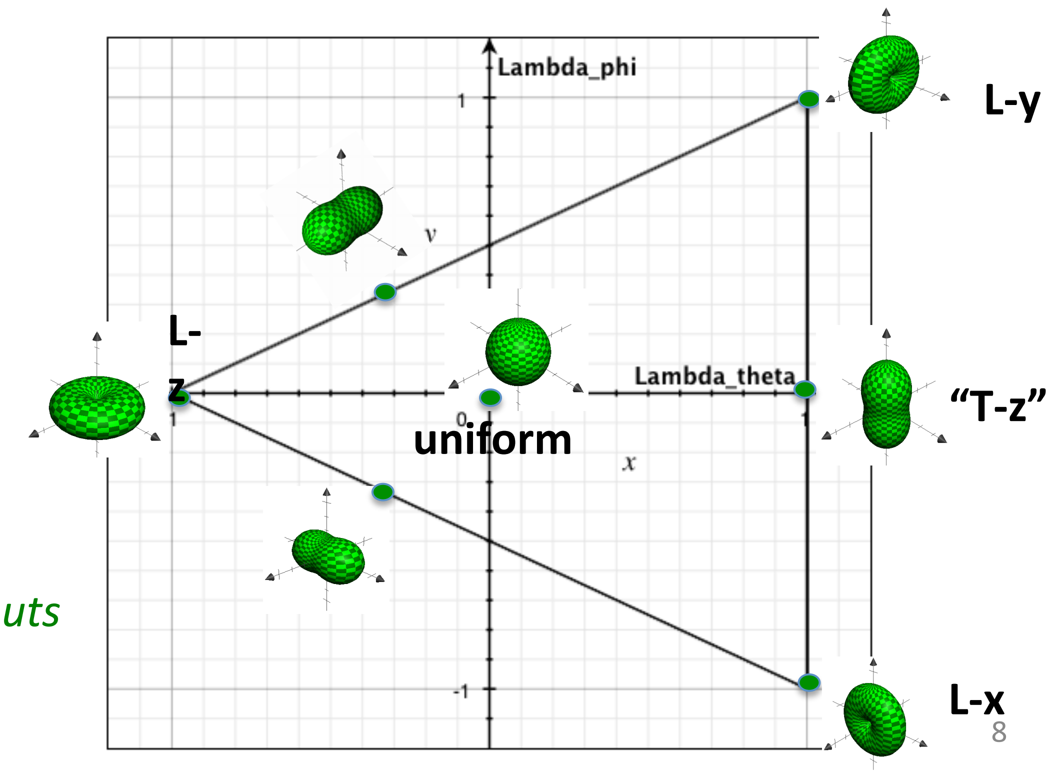
- From general principles, in any frame:

$$|\lambda_\theta| \leq 1\,, \qquad |\lambda_\phi| \leq \tfrac{1}{2}(1+\lambda_\theta)\,.$$

- Also, for any given set $[\lambda_\theta, \lambda_\varphi, \lambda_{\theta\varphi}]$ in a frame F, we could rotate to a new frame F' where $\lambda'_{\theta\varphi} = 0$

- Allowed triangle and angular distribution pattern in λ_θ, λ_φ plane ($\lambda_{\theta\varphi}$=0)

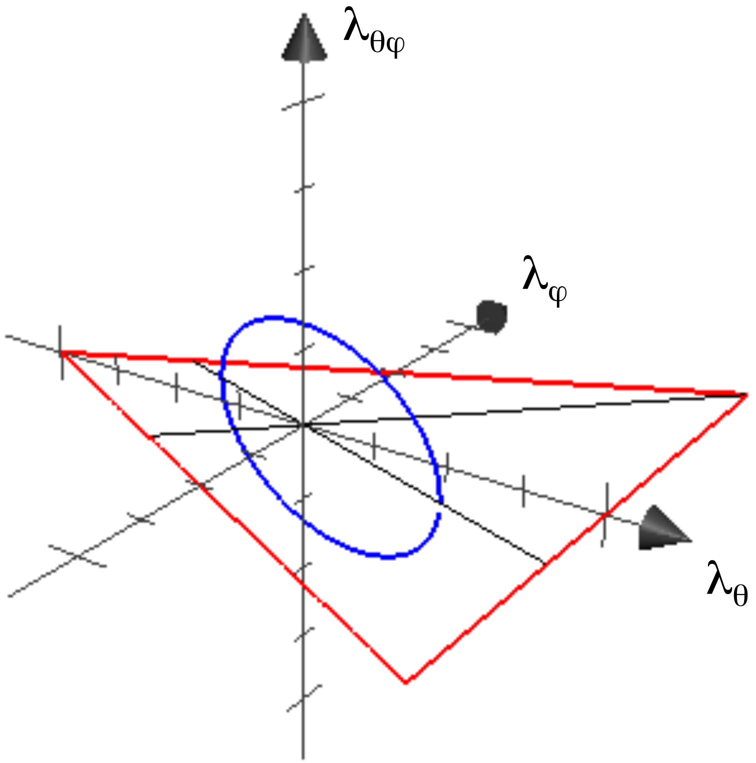
Donuts & peanuts



Rotation in the space of the coefficients

- It is convenient to consider the transformations of the coefficients of the angular distribution under rotation of frame as a geometrical rotation in the 3D space of the coefficients:

The transformations are described as ellipses in the space of the coefficients, normal to the plane $\lambda_\theta\lambda_\varphi$, and wound about the line $\lambda_\theta=\lambda_\varphi,\ \lambda_{\theta\varphi}=0$.



– A rotation by $\pi/2$ (typical between CS and H frame) links opposite points on the ellipses.

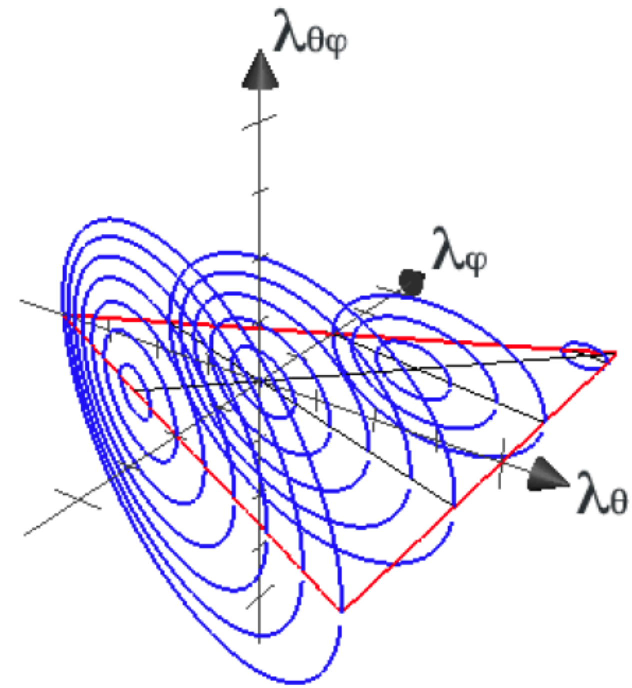
– The ellipses fill a conical volume
  (we are dealing with inclusive processes …)

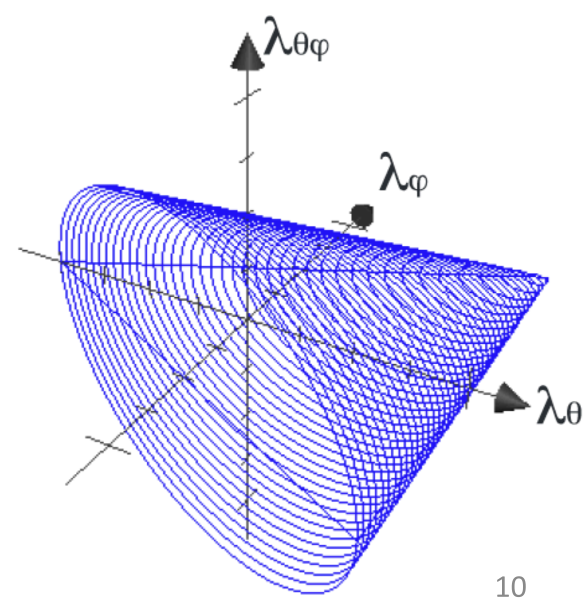
– The conical surface defines the maximum range allowed by for $|\lambda_{\theta\varphi}|$ for any given values of λ_θ, λ_φ :

$$|\lambda_{\theta\phi}|^{\max} = \frac{1}{2}\sqrt{(1-\lambda_\theta)(\lambda_\theta+1-2\lambda_\phi)}$$

– This expression completes the bounds/consistency relations between coefficients, together with:

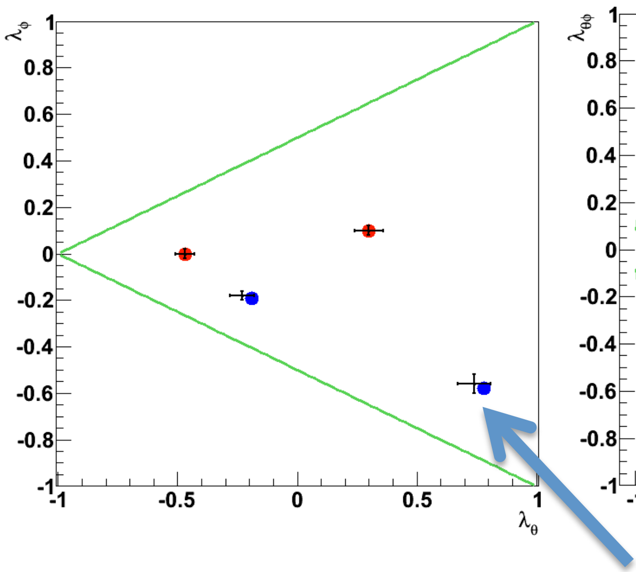
$$|\lambda_\theta| \leq 1\ , \qquad |\lambda_\phi| \leq \tfrac{1}{2}(1+\lambda_\theta)$$

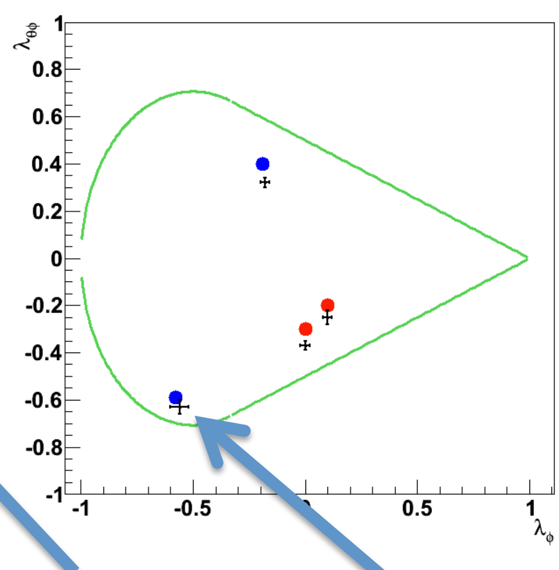


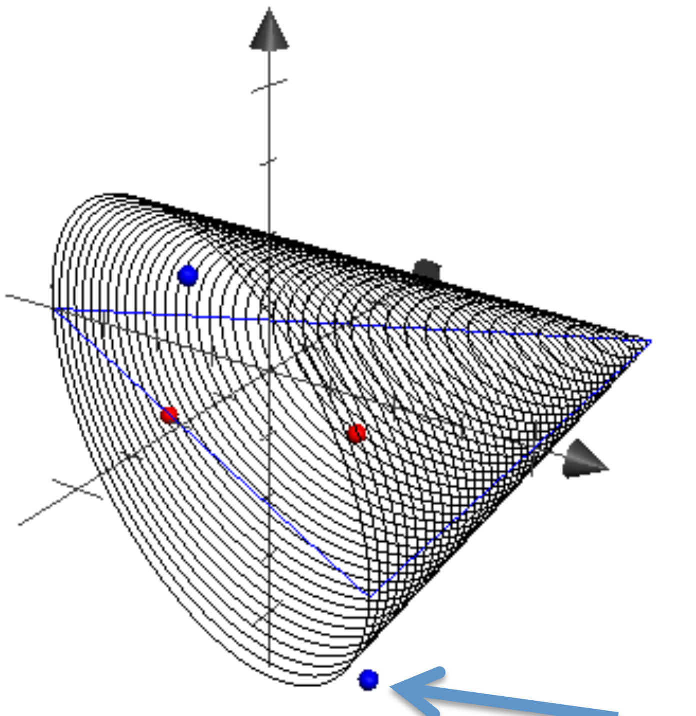


Bounds/consistency relations between coefficients

- The *conical* bound is tighter than the bounds usually shown.







Invariant combinations of the coefficients

- Relevance of invariants discussed yesterday
- An invariant may be seen as related to a property of an ellipsis, rather than to a point that loops along the ellipsis as frames are rotated.

First example: related to known invariant $\tilde{\lambda}$

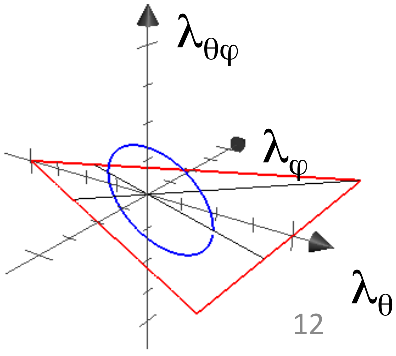
The plane containing any ellipsis defines a correlation between λ_θ and λ_φ :

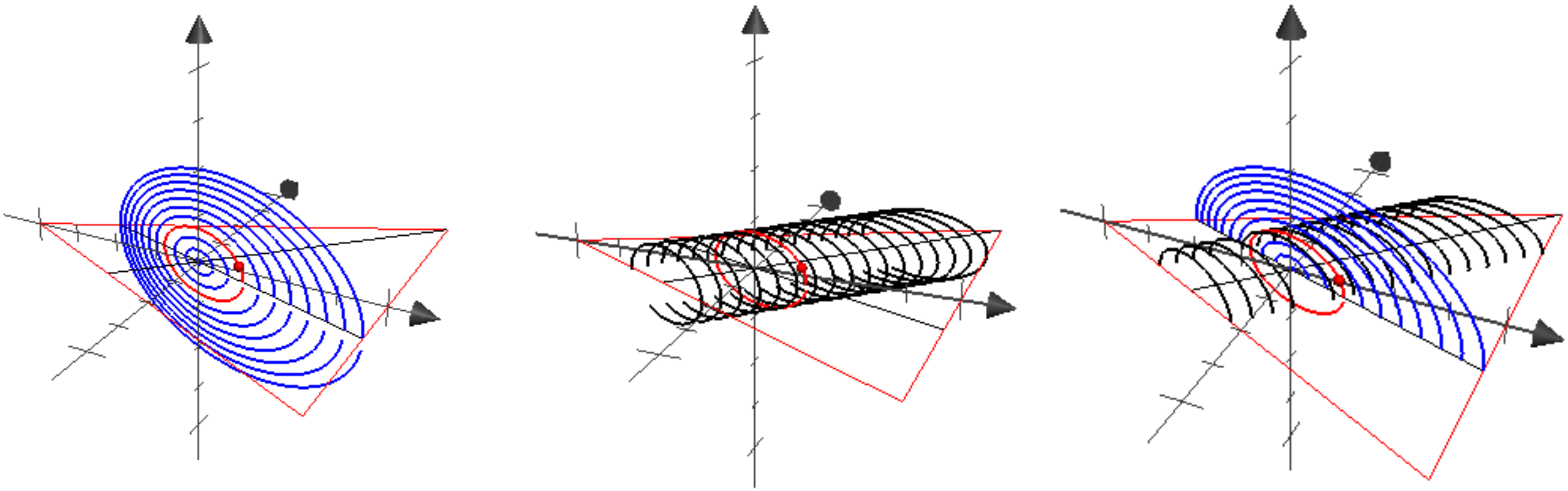
$$\lambda_\phi - 1 \;=\; -\,(\lambda_\theta + 3)/(3 + \tilde{\lambda})\;,$$

in other words: the value of $\tilde{\lambda} = (\lambda_\theta + 3\lambda_\varphi)/(1-\lambda_\varphi)$ is invariant.

A new invariant may be related to other properties, such as size of axes of ellipsis, or the range covered by the coefficients. They involve the third coefficient $\lambda_{\theta\varphi}$. An example is:

$$\lambda^* = \frac{1 + (\lambda_\theta - \lambda_\phi)/4}{\sqrt{(\lambda_\theta - \lambda_\phi)^2 + 4\,{\lambda_{\theta\phi}}^2}}$$





- $\tilde{\lambda}$ specifies a plane, λ^* specifies a cylinder (elliptical)
- Each of them alone does not specify the *intrinsic angular distribution* (but provides a relevant information)
- Together, they specify the intrinsic angular distribution
- The phase along the ellipsis specifies how the intrinsic angular distribution is oriented relative to (*e.g.*) the H and the CS frames

Summary

- A geometrical description for the transformation of the coefficients of the angular distributions under rotations of frames (in the production plane).
- Better bounds/consistency relations between the coefficients.
- A new discussion of invariants, a new invariant depending on the three coefficients of the angular distribution.

Acknowledgements and references:

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- P. Faccioli, C. Lourenço, J. Seixas and H.K. Woehri, PRL 105, 061601; PR D81, 111502; EPJ C69, 657.
- S.P., PR D83, 031503 (arXiv:1012.2485).
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